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Is Logic the Science of the Laws of Truth?

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1 Frege's Characterization of Logic

In a paper published in 1918, 'Der Gedanke. Eine logische Untersuchung' ('Thought. A Logical Investigation'), Frege says (p. 30, my translation): 'As the word "beautiful" to esthetics and the word "good" to ethics, thus the word "true" is pointing out a direction to logic. All sciences have truth as their aim, logic, however, is, in addition, involved with truth in a quite special manner. ... It is the task of logic to cognize the laws of truth [*literally*: the laws of being true].' Frege makes this more precise. He distinguishes *normative* laws from *descriptive* laws. The laws of truth are, like the laws of nature, descriptive laws, although, of course, they can be taken as a basis for prescriptions, and thus as a basis for normative laws: for the laws of *rationally correct* thinking. The laws of truth must, in turn, according to Frege be distinguished from the laws of nature; the latter describe the invariant form of what is happening in time; the laws of truth, however, describe timeless *being*. Still fighting psychologism, Frege strongly emphasizes that logic is not concerned with the *psychological* laws of factual thinking; the task of logic is to discover the laws of truth, not the empirically descriptive laws of actually occurring believing or thinking (p. 31).

Frege arrives at a general characterization of what the laws of truth consist in (p. 31, my translation): 'In the laws of truth the meaning of the word

''true'' is unfolded.' However, it seems that there is not much of interest to be expected from an investigation of *such* laws of truth, for a few lines further down Frege declares (p. 32) that any attempt to define truth [*literally*: being true] is bound to fail, due to circularity, since, according to Frege, the concept of truth must already be given - given as a primitive - in order to decide whether the allegedly truth-defining concepts apply in any considered case: for the question whether they apply is the question *whether it is true* that they apply. He comes to the conclusion (p. 32, my translation): 'It is likely that the content of the word ''true'' is totally singular and indefinable.'

In a much earlier manuscript - the fragment 'Logik' from 1897 - Frege had already arrived at practically the same positions.[1] There he writes (p. 39, my translation): 'Truth apparently is something so primitive and simple as to be not reducible to anything yet simpler.' And: 'Logic is the science of the most general laws of truth [*literally*: of being true].' Frege adds apologetically: 'Perhaps one finds that one cannot think of anything totally determinate that is meant by these words. Inaptness of the author and of language may be the cause of this. But the intention was only to give a rough characterization of the aim [of logic]. What is missing must be supplemented by the carrying out [of logical investigations].' Now, Frege certainly carried out many logical investigations where he clarified many things in and around logic; but it can hardly be said that his very characterization of logic as *the science of the laws of truth* has been clarified by these investigations further than it is explicated by his rather scant remarks that he explicitly attaches to the characterization. At least, however, in the course of his logical investigations, Frege amply provided us with examples of what he thought *are* laws of truth. But are those examples indeed what Frege thought they are? Are even the axioms and theorems of a classical propositional calculus laws of truth? Note that if logic is to be the science of the laws of truth, it is not enough that those axioms and theorems can in some sense also be construed to be laws of truth. No, they must be primarily and essentially laws of truth - albeit that need not be obviously or trivially so.

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[1] Already in the introduction to the first volume of *Grundgesetze der Arithmetik* (*Fundamental Laws of Arithmetic*) published in 1893, we find Frege asserting that logical laws are laws of truth (see p. XVI). But there (pp. XV-XVII) the attack against psychologism is paramount (logical laws, says Frege, are laws of *being true*, and *not* natural laws of *holding to be true*), and the definition of logic as the science of the laws of truth is only implicit, but nevertheless unmistakably implied. Thus, in the *Grundgesetze*, we find that definition coupled with Frege's assumption of having faultlessly shown (p. XXVI) that 'arithmetic is nothing but further developed logic' (p. VII) - *without* Frege being aware that there might be a problem in this coupling. See for this Section 1.4 below.

1.1 ... and Modern Logic

But let us first, before tackling the question just posed, look beyond the confines of classical Fregean logic. Let us consider logic with the very large scope it has acquired today. Logic in its modern form, as currently practiced, is certainly *not* the science of the laws of truth, nor is its relation to truth in any way special. Of course, like all sciences, modern logic too is aiming at truth, namely, like each and every science, at the particular truths it is interested in, for example: Is it true that formula F is provable in system S? Is it true that system S is deductively equivalent to system S'? Is it true that system S is sound and complete with respect to the model structures in set M? Indeed, there are many truths modern logic is interested in, but none of them is a law of truth in approximately the sense Frege seems to have in mind.

Moreover, modern logic is certainly interested in the axioms and theorems of logical calculi (as has just been said, at least some of those formulas are intended by Frege to be laws of truth); but modern logic is not interested in them *qua* laws of truth. In the first place, the word 'true' does not even occur in them. But this is not a serious objection, since we can insert 'it is true that' in front of any sentential part of any formula without changing its meaning. The second objection is decisive: Modern logic proceeds fairly close to the maxim: 'Give me the axiomatic system, *whatever it is*, and I give you its models (if there are any).' Or vice versa: 'Give me the models, *whatever they are*, and I give you the axiomatic system for them (if there is one).' Take a tense-logical principle like $Fp \supset FFp$. This formula (expressing the density of the temporal ordering) is logically valid with respect to an appropriate set of model structures. But it is not logically valid with respect to a certain other set of model structures; with respect to that set we find, on the contrary, the discreteness-formula $Pq \wedge p \wedge \neg Fp \supset P(Fp \wedge \neg FFp)$ to be logically valid. The modern logician is happy to formulate two alternative systems, one in which $Fp \supset FFp$ is provable, and one in which it isn't provable, but $Pq \wedge p \wedge \neg Fp \supset P(Fp \wedge \neg FFp)$ is. Thus, although modern logicians are interested in formulating axiomatic systems, they are *qua modern logicians* not interested in assuring that those systems codify laws of truth or even *something true*. The truth *of* their axiomatic systems - contrary to the truth *about* them - does not matter to modern logicians *qua* modern logicians. It is enough if some interest attaches to the provable formulas - some interest derived from some envisaged application - be it artificial intelligence, the semantics of natural language, or even merely some puzzling argument in the history of philosophy.

1.2 ... and Logic as a Branch of Philosophy

From the modern point of view, Frege's characterization of the aims of logic is hopelessly outmoded and sounds quaintly philosophical. Although logic, alongside psychology, was the very last fledgling to leave the nest of philosophy, logic is today very much independent of its mother. Modern logicians do not much care about finding the laws of truth (or even the normative laws of correct reasoning); if logic is not practiced for its own sake (as the pure science of formal symbolic systems), it has - like applied mathematics - become a discipline of abstract engineering, catering indiscriminately to various special needs, among them those of philosophers. In the latter case, it is indeed called 'philosophical logic'; but this must not blind one to the fact that there is nothing intrinsically philosophical about so-called 'philosophical logic'.

If we take Frege in saying that *logic is the science of the laws of truth* as wanting to explicate the word 'logic', then our criticism will have to be that his explication departs too far from the *presently given* pre-explicatory, but nevertheless informed standard usage of the word, and is therefore not adequate according to the standards of adequacy for explications laid down by Rudolf Carnap. But this would certainly be unfair to Frege; for Frege intends to explicate the word 'logic' under the restraint that logic *has* to be a branch of philosophy. He took logic to be necessarily a philosophical discipline, a part of philosophy. Thus he is, rightly considered, not explicating the word 'logic', but the words 'logic as a branch of philosophy'. In saying that logic is the science of the laws of truth, Frege must today be taken to say that logic *as a branch of philosophy* is the science of the laws of truth; for him, the tag 'as a branch of philosophy' was simply redundant.

It is not *for us*. But what Frege *meant* to say may still be correct. There certainly exists a part of present-day logical activity - a small part, and unfortunately, it seems to me, a more and more diminishing one - which is at the same time a part of philosophy. Due to tradition - in the past all of logic was a part of philosophy - this small part of the sum total of present-day logical activity is still also simply called 'logic'; but nowadays it would serve clarity to call it 'logic as a branch of philosophy'.

1.3 ... and Classical vs. Non-Classical Logic

So it may be correct, as Frege holds, that logic - if taken as a branch of philosophy - is the science of the laws of truth. But *is* it in fact correct? Certainly not uncontroversially. Philosophically-minded logical intuitionists hold that logic is the science of the laws of *provability* or of *justified assertability*. From their point of view, logic could be the science of the laws of truth only if 'true' were co-extensional with 'provable' or with 'justifiedly assertable'. But, in the normal sense of the word, 'true' is certainly not co-extensional with 'provable' or with 'justifiedly assertable', and it seems to

me that Frege and most other people would wish to hold on to this aspect of the normal sense of the word 'true'. So would most logical intuitionists, but they would add that they do not find the concept of truth - as distinguished from provability and justified assertability - to be a particularly useful one; they would urge us to get rid of it. For them, the axioms and theorems of their restricted - hence non-classical - calculi, e.g., of propositional logic, cannot be primarily or essentially laws of truth, and hence, for them, logic cannot be the science of the laws of truth.

Thus it is by no means philosophically uncontentious that logic is such a science. But again, it would be unfair to hold this against Frege's definition. For Frege is clearly explicating the word 'logic' under the further restraint that logic - as a branch of philosophy - is identical to *classical* logic. Like the restraint that logic must be a branch of philosophy, the restraint that logic is identical to classical logic was unrecognizable to Frege. The possibility that logic might offer alternatives to classical logic did not occur to him, just as the possibility did not occur to him that logic could be more than a branch of philosophy. Thus, when Frege says that *logic is the science of the laws of truth*, he must today be taken so say that *classical logic - as a branch of philosophy - is the science of the laws of truth.*

1.4 ... and Logicism vs. Elementary Logic

Obviously, Frege's explicatory thesis about the nature of logic is gaining in plausibility, once the restrictions are made explicit that Frege tacitly, and indeed unconsciously, deposited into its meaning. In fact, by now Frege's thesis seems so plausible as to appear trivial. Doesn't classical logic center on truth and falsity, where these two concepts are, in addition, obeying the principle of bivalence (or in other words, where each of these concepts is the negation of the other)? But even if it is so, is this enough to make classical logic, even within philosophy, the science of the laws of truth? For are such laws the *primary* objects of cognition for classical logic, or rather something else? Only if such laws are indeed the primary objects of cognition for classical logic, and not merely, as it were, by-products of its activities, can classical logic be indeed the science of the laws of truth.

At this point we should remember that Frege was also a philosopher of mathematics and went about - in the *Grundgesetze der Arithmetik* (*Fundamental Laws of Arithmetic*) - realizing the logicistic program of reducing arithmetic to logic. But the primary objects of arithmetical cognition are surely not laws of truth, but abstract structures: numbers and systems of numbers. Hence, from the point of view of logicism, it is incongruent to regard laws of truth as the primary objects of logical cognition. If logicism is right, logic - that is, for Frege, classical logic, a branch of philosophy - must at least sometimes be primarily concerned with other things than with laws of truth - namely with the object-related truths about certain 'logical

objects', as Frege calls them. This difficulty for Frege's definition of logic, which arises from the very aspirations he had for logic, points us, however, to yet another restriction Frege can be taken to be tacitly assuming when he is speaking of logic as being the science of the laws of truth: At least later in his career, by the word 'logic' Frege may very well be merely intending *basic predicate logic*, which of course includes (truth-functional) propositional logic.

This further restriction is strongly suggested by the contents of the latest text in which his definition of logic occurs, the *Logische Untersuchungen* (*Logical Investigations*), published in three installments between 1918 and 1923 (I have quoted from the first installment above in Section 1). In the three published parts of *Logische Untersuchungen* Frege is *exclusively* concerned with central aspects of classical propositional logic. A fourth part, dedicated to classical predicate logic, and which would presumably have been mainly, if not merely, concerned with its more elementary aspects, was begun by Frege, but not finished. Since in the fragment of the fourth part he refers to the third part (which bears the title 'Logische Untersuchungen. Dritter Teil: Das Gedankengefüge' - 'Logical Investigations. Third Part: The Thought-Structure') as already published, Frege's work on the *Logische Untersuchungen* was apparently cut short only by his death in 1925.[2]

In order to give Frege's explicatory definition of logic the greatest chance of being correct or adequate, we may take him to be *ultimately* saying, in saying that logic is the science of the laws of truth, that classical *elementary* logic - as a branch of philosophy - is the science of the laws of truth. Indeed, at the end of his life, he *could* well have arrived at the conclusion that even if arithmetic is a part of some science which is a branch of philosophy, that science cannot properly speaking be called 'logic'.[3] And Frege would have been quite right in thinking so. Any science in which it is asserted (be it as an axiom or as a theorem) that there are infinitely many objects - a strange law of truth indeed! - cannot, properly speaking,[4] be called 'logic', no matter whether that assertion refers to a universe of objects which is divided into hierarchically ordered layers, or to one that does not have this hierarchical structure that is so satisfying to the order-loving

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[2] See the introduction by Günther Patzig, the editor of *Logische Untersuchungen*, pp. 5-7.

[3] In the fragment 'Neuer Versuch der Grundlegung der Arithmetik' ('New Attempt to Found Arithmetic') from 1924/25 he begins to attempt a geometrical foundation of arithmetic. But he also makes two striking statements (p. 298, my translation): 'I had to give up the opinion that arithmetic is a branch of logic and that therefore in arithmetic everything has to be proved purely logically' and (p. 299) 'It seems that the logical source of cognition cannot by itself present us with objects'.

[4] 'properly speaking', that is: without missing the entrenched part of the meaning of the word.

minds of modern proponents of logicism.[5] And therefore, since arithmetic requires an infinity of objects, a science which is able to incorporate arithmetic cannot, properly speaking, be logic. If that science *is* to be a branch of philosophy, it must be *ontology*, or in other words: *general metaphysics*.[6]

1.5 ... and Frege's Elementary Oversight

Ironically, Frege's logicistic program - the high aspirations he had for logic - may be said to have failed precisely because Frege did not pay enough attention to the results of classical elementary logic, that is, to precisely those homely laws of truth he apparently in the end meant to be all that logic is concerned with. The following is a theorem-schema of elementary predicate logic: $\neg\exists y\forall x(R(x,y) \equiv \neg R(x,x))$. Russell's letter to Frege in which Russell formulated his famous antinomy, in effect, merely pointed out to Frege that he had violated this very law of truth, thus making his system in the *Grundgesetze der Arithmetik* logically inconsistent. Never before had a logician been so tragically punished as a logician for having been blinded by a metaphysical hope! For Frege, this hope - to which he entirely dedicated himself without realizing its metaphysical, non-logical character - had been the *ontological* reduction of arithmetic to his *ontological* theory of functions and objects, and in particular *Wertverläufe* (*courses of values*). For the realization of this hope he threw away a logician's carefulness and wariness. I do not see how otherwise the inconsistency of his system could have remained hidden to him for such a long time.

1.6 ... and Intensionalism in Logic

But can we accept Frege's ultimate, modest claim that classical elementary logic as a branch of philosophy (and therefore *without* the heavy concentration on proof-theoretic and model-theoretic issues which is so characteristic of modern logic and which has almost entirely replaced interest in the contents of the logical systems themselves)[7], that logic *thus* regarded (I from now on leave tacit the modifier 'as a branch of philosophy') is the science of the laws of truth?

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[5] Cf. e.g. Franz v. Kutschera who in Kutschera 1985 takes Zermelo-Fraenkel axiomatic set theory to be logic (p. 225) since it is provably equivalent to the overtly hierarchical (or 'inductive' - cf. pp. 210-212) system of Dana Scott (pp. 213-215, pp. 223-224).

[6] Frege *could* have arrived at this view - I am not saying that he did. Indeed it is rather unlikely that he ever realized the metaphysical nature of his so-called logicism. The reason for this is the predominant Neo-Kantian anti-metaphysical atmosphere in Germany in the second half of the 19th century: Metaphysics and ontology were banished from philosophy, and any self-respecting philosopher, imbued by that atmosphere, could not allow himself to regard his position as ontological and his practice as ontology.

[7] I am not saying that these issues are without philosophical interest. On the contrary, some results in this area are of the greatest philosophical importance *for epistemology*: the undecidability of predicate logic, the incompleteness of axiomatized arithmetic.

Since the scope of classical elementary logic is certainly too small to capture *all* laws of truth (laws of truth involving modalities, for example, are outside its ken), we have to replace the phrase 'the laws of truth' by the phrase 'the *most basic* laws of truth', as is indeed suggested by Frege himself: In the fragment 'Logik' from 1897 he had said (see the quote in Section 1 above) that logic is the science of the *most general* laws of truth. Presumably, what is most general is most basic, and vice versa.

Then there still remain two nagging closely related questions which we have already encountered in a modified form on the way to the present position: (1) Are the objects of cognition of classical elementary logic *primarily* laws of truth? (2) Are the axioms and theorems of a system of classical elementary logic - and in particular those of a system of classical propositional logic - *primarily* laws of truth? Since the axioms and theorems of a system of classical elementary logic are precisely the primary objects of cognition of classical elementary logic, the first question reduces to the second; this question is what we now must concern us with.

To a student of logic who, in the spirit of Frege, has been taught the meaning of the classical sentence-connectives 'and', 'or' and 'not' by truth-tables and who has then been made familiar with the utterly simple truth-value semantics for classical elementary predicate logic - a semantics that does without a universe of discourse - the answer to question (2) may seem to be obvious: 'Yes, the axioms and theorems of a calculus of classical elementary logic *are* primarily, essentially and naturally laws of truth, and indeed laws of truth without any ontological content whatsoever, albeit, *pace* Frege, they certainly contain more than a mere development of the meaning of the word "true".' However, even in Frege's own works we can find a massive hint that the answer 'Yes' to the question at issue is not necessarily correct. In the 'Ausführungen über Sinn und Bedeutung' ('Expositions on Sense and Reference', written between 1892 and 1895) we read (pp. 31-32, my translation): 'The logicians of content like only all too much to stop at the sense [the meaning]; for what they are calling content is, if not perception, so certainly sense. They do not consider that what matters in logic is not how thoughts [propositions] result from thoughts, without respect to the truth-value; they do not consider that the transition from the thought to the truth-value and - generally speaking - from the sense to the reference [*literally, but misleadingly*: the meaning] must be made, that the logical laws are primarily laws in the realm of reference and only secondarily pertain to sense.' Frege does not explain why the necessity of making the transition from the sense to the truth-value requires that logical laws are *primarily* laws in the realm of reference. Even if we agree with Frege that the truth-value of a sentence is its reference, there is no necessary connection between the two positions. Rather, a 'logician of content' - or in other words: a proponent of *intensionalism* in logic - can agree with Frege that

the transition from the thought to the truth-value, from the sense to the reference *must* be made, and yet deny that the logical laws are primarily laws in the realm of reference. For an intensionalistic logician, logical laws are primarily laws in the realm of sense, and only *secondarily* do they pertain to reference and hence to truth.

2 The Intensionalistic Characterization of Logic

It follows that for an intensionalistic logician the axioms and theorems of a calculus of classical elementary logic are neither primarily nor essentially laws of truth, *although* their applicability in the realm of truth and reference is of course a highly important reason for our taking any interest at all in those axioms and theorems. Thus, regarding them as laws of truth is undoubtedly a natural, but nevertheless *secondary* perspective on them. If this secondary perspective is made the primary one (as is done by Frege), then, for the intensionalistic logician, this is comparable to declaring that the principles of physics are *primarily* laws for the building of rockets, televisions, and nuclear weapons. Thus, even if intensionalistic logicians accepted the highly restricted Fregean range of the word 'logic', they would certainly deny that logic is the science of the most basic laws of truth; this denial would be as obvious to them as denying that physics is the science of the most basic laws of technology is obvious to physicists. For the intensionalistic logician *logic is the science of the most basic meaning relations.*

3 Intensionalistic Logical Validity and Consequence

Is Frege right, or the 'logicians of content'? I will demonstrate for the simple case of classical propositional logic that the cause of the intensionalists is a strong one, making good the expectation Adolf Reinach voiced in his 1911 paper 'Zur Theorie des negativen Urteils' ('On the Theory of the Negative Judgment') (p. 251, my translation) 'that large parts of traditional logic will in their foundations prove to be general proposition theory [allgemeine Sachverhaltslehre]'.[8]

I have already mentioned that there is no mention of truth in the axioms and theorems of logical systems. Much more importantly, there need not be any mention of truth in the semantics for such systems: The fundamental semantic concepts of *logical validity* and *logical consequence* can be intuitively satisfactorily defined without making any use of the concept of truth at all. All of this is very well known under the heading of *algebraic seman-*

[8] If the foundations of logic are in question, it is more appropriate to translate Reinach's term 'Sachverhalt' by 'proposition' than by 'state of affairs'. Behind the intensionalist Reinach there stand the earlier intensionalists Husserl and Bolzano (as Wolfgang Künne emphasized in discussion).

tics; but let me rehearse that part of it that concerns classical propositional logic in order to show that even there Frege's definition of logic is not incontestable:

(1) Let L be a language of propositional logic, built in the usual manner on the basis of an infinity of propositional variables p, p′, p′′, etc. by applying the sentence-connectives $\neg$, $\wedge$, $\vee$, and brackets.

(2) An *interpretation* of L is a pair <W,V> consisting of a non-empty set of worlds W and a function V which assigns to every propositional variable of L a subset of W, and to all other formulas of L a subset of W according to the following stricture: For all formulas φ and ψ of L: $V(\neg\varphi) = W-V(\varphi)$, $V(\varphi\wedge\psi) = V(\varphi)\cap V(\psi)$, $V(\varphi\vee\psi) = V(\varphi)\cup V(\psi)$.

(3) φ is an L-logical consequence of $\psi_1,...,\psi_n := \varphi$, ψ_1, ... ,ψ_n are formulas of L, and for all interpretations <W,V> of L: $V(\psi_1)\cap...\cap V(\psi_n)\subseteq V(\varphi)$.

(4) φ is L-logically valid := φ is an L-logical consequence of $\neg\varphi$.

Intuitively, (3) says that a formula φ of L is a logical consequence of formulas $\psi_1,...,\psi_n$ of L if and only if the proposition expressed by φ is *intensionally contained* in the proposition the conjunction of $\psi_1,...,\psi_n$ expresses, *no matter* which particular propositions are expressed by φ and $\psi_1,...,\psi_n$. Thus (3) encodes an intensionalistic conception of logical consequence, a conception which the 'logicians of content' would undoubtedly favor and which undoubtedly is intuitively at least as satisfactory as the orthodox extensionalistic conception.[9] Note that there is no mention of truth in the definiens of (3); all that is talked about is the *propositional contents* of certain formulas and how those contents are related. Given (3), it is clear that in (4) also the logical validity of a formula of L is defined merely in terms of the propositional contents of certain formulas and how those contents are related. There is no need whatever of the concept of truth. Nevertheless, the very same formulas and inferences of L turn out to be logically valid that are logically valid according to the orthodox extensionalistic semantics for L, and these formulas, axiomatized by some adequate calculus or other, are precisely those that form the canon of classical propositional logic. Thus we have a clear demonstration that the laws in this canon have no unseparable relation to truth. They are not essentially laws of truth, and, in view of the

[9] Cf. the following remarkable passage from Reinach's 'Zur Theorie des negativen Urteils' (p. 222, my translation): 'All connections of justification encountered by us in science or in daily life are connections of propositions. This is also true of the connections that are generally subsumed under the name of "laws of inference": They are, rightly regarded, nothing else but general nomological relations of propositions. The fundamental consequences that result from this insight for the construction of the science of logic are obvious.'

intuitive satisfactoriness of definitions (3) and (4), it is more than doubtful whether they are at least primarily or naturally laws of truth.

4 Intensionalism with Truth Not Ignored

One might object that the intensionalistic conception of propositional logic is ontologically costly: Even if intensionalists who want to do propositional logic can get around assuming a plurality of possible worlds (in order to construe coarse-grained propositions simply as sets of worlds), they surely cannot get around positing propositions in some sense, together with an appropriate propositional algebra, as additional entities. To this I can merely say: So what? What is bad about propositions? And of course the ontological-economy-objection with respect to propositions is no objection Frege either would or could have raised. He was not an ontological scrooge. Ontological miserliness was not one of his vices, and, as we all know, his ontology comprises what he called *thoughts*, entities that, notwithstanding the word Frege used to designate them, have nothing to do with subjectivity, but are what we would call *fine-grained propositions*.

Indeed, it is hard to see what Frege could answer to a modern intensionalistic logician, contesting in the way presented above Frege's claim that logic is the science of the laws of truth. Presumably he would insist that the transition from sense to reference, from the thought to the truth-value *must* be effected. But modern intensionalistic logicians - whatever their forebears did, who according to Frege liked only all too much to stop at the sense[10] - would be quite ready to comply with Frege and bring truth into the game - although not quite in the manner Frege had in mind.

The above semantics for classical propositional logic can be made to yield laws of truth, and indeed not merely in the trivial sense that every sentential part of a principle of classical propositional logic can, without changing its meaning, be prefixed with the words 'it is true that' (or with a symbol having the meaning of this phrase). For this purpose we re-define what is an interpretation of the language L as follows:

(2*) An *interpretation* of L is a triplet $<W,\omega,V>$ consisting of a non-empty set of worlds W, a subset ω of W, and a function V which assigns to every propositional variable of L a subset of W, and to all other formulas of

[10] Adolf Reinach, at least, is a notable exception to Frege's allegation. Shortly before he (for the second time: cf. Section 3) envisions large parts of traditional logic as being founded on general proposition theory, he says ('Zur Theorie des negativen Urteils', p. 251, my translation): 'A sentence is true if the corresponding proposition obtains. And two contradictory sentences cannot both be true *because* two contradictory propositions cannot both obtain. Thus, also in this case the sentence-law leads back to a proposition-law. At the same time, we have here an example that illustrates in what sense we asserted above that large parts of traditional logic will in their foundations prove to be general proposition theory.'

L a subset of W according to the following stricture: For all formulas φ and ψ of L: $V(\neg\varphi) = W - V(\varphi)$, $V(\varphi \wedge \psi) = V(\varphi) \cap V(\psi)$, $V(\varphi \vee \psi) = V(\varphi) \cup V(\psi)$.[11]

We can then define the concept of *truth in an interpretation* for formulas φ of L as follows:

(5) φ is true in $\langle W,\omega,V\rangle$:= $\langle W,\omega,V\rangle$ is an interpretation of L, and $\omega \subseteq V(\varphi)$.

The intuitive meaning of this definition is the following: A formula φ of L is true (in an interpretation $\langle W,\omega,V\rangle$ of L) if and only if the proposition that φ expresses (in that interpretation) is intensionally contained in the proposition which is the intersection (or *intensional sum*) of all *obtaining* propositions (for that interpretation). If we want to derive, on the basis of (5) and (2*), the classical truth-conditions for the propositional connectives, and indeed all the *metalinguistic* laws of truth one would normally assume for them (*these* metalinguistic laws, not certain object-language formulas, really deserve the designation '[logical] law of truth'!), then we need to add one extra condition for ω to (2*): ω *is a singleton set* (in other words: ω can be reduced to precisely one possible world in W). If we allow ω to be empty, then we cannot prove: *For all interpretations* $\langle W,\omega,V\rangle$ *of* L *and formulas* φ *of* L: φ *or* $\neg\varphi$ *is not true in* $\langle W,\omega,V\rangle$. And if we allow ω to have more than one element, then we cannot, for example,[12] prove: *For all interpretations* $\langle W,\omega,V\rangle$ *of* L *and formulas* φ *of* L: φ *or* $\neg\varphi$ *is true in* $\langle W,\omega,V\rangle$. But already if we simply take (2*) as it is, without any extra condition for ω, then we can prove *for all and only* the principles π of classical propositional logic (as formulated in L and as codified in an appropriate axiomatic system): π *is true in every interpretation* $\langle W,\omega,V\rangle$ *of* L.

5 What is Primary: Intensional Containment or Truth?

The upshot of this is that Frege could not accuse the modern 'logicians of content' of disregarding the importance of truth for logic. Nevertheless, laws of truth, such as those presented above, are for the intensionalistic logician no more than a most welcome side-effect of logic, which, however, is *primarily* concerned with laws of meaning relations.

Whether Frege's view of logic is to be preferred or that of the intensionalists at this point crucially depends on the question whether the central concept of the intensionalists, the *intensional containment* of one proposi-

[11] Additional clauses can be added for *modal extensions* of L: $V(\Box\varphi)=W$, if $V(\varphi)=W$, and $V(\Box\varphi)=\emptyset$, if $V(\varphi)\neq W$; $V(\Box^*\varphi)=W$, if $\omega \subseteq V(\varphi)$, and $V(\Box^*\varphi)=\emptyset$, if not $\omega \subseteq V(\varphi)$.

[12] Another law of truth that is not provable if ω has more than one element is the following: *For all interpretations* $\langle W,\omega,V\rangle$ *of* L *and formulas* φ *and* ψ *of* L: *if* $\varphi \vee \psi$ *is true in* $\langle W,\omega,V\rangle$, *then* φ *or* ψ *is true in* $\langle W,\omega,V\rangle$.

tion by another, can be understood without even tacitly presupposing the concept of truth. Now, the proposition *that the apple is colored* intensionally contains the proposition *that the apple is extended.*[13] Does this need any gloss in terms of truth? For example the gloss: the sentence 'the apple is colored' cannot be true without the sentence 'the apple is extended' being true? Or the alternative gloss: it cannot be true that the apple is colored without it being true that the apple is extended? It seems to me the intensionalists would be within their rights if they asserted the concept of intensional containment to be primitive and sufficiently clear, and if they added: 'The sentence ''the apple is colored'' cannot be true without the sentence ''the apple is extended'' being true *because* the proposition that the apple is colored intensionally contains the proposition that the apple is extended, and the direction of explanation *is not* the other way round, as is groundlessly presumed by the extensionalists.'

6 Frege and Three Uses of 'True'

Since Frege employed the concept of proposition (under the name of 'thought') and believed in the existence of propositions, Frege - even given the paramount importance he accorded to truth in logic - might have become reconciled with logical intensionalism, *if* he only could have seen a viable method of defining truth in an intensionalistic framework.[14] But he never succeeded in seeing *any* method of defining truth, he never even succeeded in seeing truth as an analyzable property. Frege really did not have any very clear idea of truth, and hence of laws of truth, at all.

The reason for this is that Frege never clearly distinguished the two fundamental and fundamentally different object-language uses of the word 'true' from each other (he *did* distinguish from them the *metalinguistic* use, where 'true' is applied as a predicate to sentences, but considered it *secondary* and did not pay much attention to it)[15]: the *ontological* use where 'true' is applied as a predicate to propositions, and that use where 'true' merely functions as a semantically redundant monadic sentence-connective (but is employed in order to make an assertion more emphatic). Listen to

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[13] Both propositions concern the same apple and the same moment of time.

[14] Indeed (as Gottfried Gabriel pointed out in discussion) there are unmistakable intensionalistic leanings in Frege's last published work *Logische Untersuchungen*, especially in its third installment 'Gedankengefüge', where the connectives 'and' and 'not' are clearly taken to express *propositional* functions (functions that form propositions - *thoughts* - from propositions) - cf. pp. 72-73 - and *true and false as logical objects* play only a minor role. Apparently Frege's final move towards intensionalism is connected with his abandoning the logicistic program (or more generally speaking: with his abandoning his belief in *logical objects*; cf. Footnote 3).

[15] See 'Der Gedanke', p. 33 (my translation): 'And when we call a sentence true, we really mean its sense.'

this passage from 'Der Gedanke' (p. 34, my translation): 'It is also remarkable that the sentence "I smell the odor of violets" has the same content as the sentence "It is true that I smell the odor of violets". Thus nothing seems to be added to the thought [proposition] by my attributing the property of truth to it. And yet, is it not a great success when after long hesitation and arduous investigations the researcher can finally say "what I surmised is true"? The meaning of the word "true" seems to be completely singular. Could it be that we are here concerned with something that cannot be called a property in the ordinary sense at all? In spite of this doubt, I will for the time being in accordance with ordinary usage continue to express myself as if truth were a property until something more appropriate will have been found.' Here Frege does not realize that it makes a fundamental difference whether the sentence 'It is true that I smell the odor of violets' is parsed as (1) 'It is true that' + 'I smell the odor of violets', or as (2) 'It is true' + 'that I smell the odor of violets'. According to the first parsing, nothing, indeed, is added to the (cognitive) meaning of 'I smell the odor of violets', nor does the first parsing exhibit an attribution of a property to a thought. According to the second parsing, however, something is obviously added to the meaning of 'that I smell the odor of violets' - after all, the sentence 'it is true that I smell the odor of violets' is not synonymous to the phrase 'that I smell the odor of violets'; in fact, the second parsing, being a *subject-and-predicate* parsing, *does* exhibit an attribution of a property to a thought: The property of being true is attributed to the thought *that I smell the odor of violets*.[16] Frege could not see this because he kept confusing the second parsing with the first. In a passage parallel to the one cited above, in the much earlier (1892) paper 'Über Sinn und Bedeutung' ('On Sense and Reference'), he already insisted (p. 49, my translation) 'that the relationship of the thought to the true must not be compared with that of the subject to the predicate'. He really did not have any good reason for holding this.

7 The Definability of 'True', pace Frege, and *the True*

All three common uses of the word 'true' - the redundant, the ontological and the metalinguistic one - are easily definable for large fragments of natural language:

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[16] Thus the sentence 'it is true that I smell the odor of violets' is indeed ambiguous: *It has two different meanings*, corresponding to the two different ways in which it can be parsed. Yet, 'it is true that <I smell the odor of violets>' must be true if 'it is true <that I smell the odor of violets>' is true, and vice versa. (But note that an ontological skeptic with respect to the existence of propositions could doubt the *vice versa*!) Hence the two disambiguated sentences resulting from 'it is true that I smell the odor of violets' stand in the same semantical relationship towards each other in which '*d* is an equilateral triangle' and '*d* is an equiangular triangle' stand: The latter two sentences have different meanings, but neither one of them can be true without the other being true.

For 'A' sentences are substitutable, for '*p*' names of propositions, for '*s*' names of sentences:

It is true that A := A.
It is true(*p*)[17] := *p* is a fact (*p* obtains, *p* is the case).[18]
('that A' - being a name for a proposition - is substitutable for '*p*'!)
s is true := the proposition expressed by *s* is a fact.

Frege presumably would have objected what was mentioned above (in Section 1): that we already need to presuppose the notion of truth in order to apply the definiens of any definition of truth in any given case. But this is not a good objection. For instead of asking ourselves *whether it is true that the definiens applies* in any given case, we can simply ask ourselves *whether it applies*. Or Frege might have objected that the above definitions offer mere synonyma for 'true'? But this can hardly be the case since they make the concept of truth clearer than it was before, and certainly clearer than it was to Frege himself.

For him, curiously, laws of truth must be laws concerning a certain *object* (*Gegenstand*), *the true*, which is a *saturated* entity and hence cannot be a property.[19] It is not amiss to see how Frege's rather unnatural (but technically useful) notion of object truth-values, of truth and falsity *as objects*, can be fitted into an intensionalistic framework: If we employ a framework of coarsely individuated propositions, then we can quite naturally identify *the true* with the tautological proposition, and *the false* with the contradictory proposition (propositions, being saturated entities, are after all objects in Frege's sense).

8 What Remains

Frege's main motivation for claiming that logic is the science of the laws of truth was his opposition to psychologism. This is quite clear from the contexts in which his claim occurs. But psychologism being long since defeated, to what uses can we put today Frege's dictum that logic is the science of the laws of truth?

Even if it surely cannot serve as a generally acceptable characterization of logic, of its subject matter and its aims, even in its more elementary and traditional parts, Frege's definition of logic certainly draws our attention again to the fact that we are interested in logic to a high degree because its application in truth-directed investigations points us from truths to further

[17] That is: '*This* is true: *p*', or more idiomatically: '*p* is true'.

[18] Not all propositions are facts. For example, that Munich is the capital of Germany in 1998 is not a fact (this is much better than saying that it is a *non-existent* fact), and therefore, according to definition, that Munich is the capital of Germany in 1998 is not true.

[19] See 'Über Sinn und Bedeutung', pp. 48-49.

truths. This, certainly, is why money ought to be spent for logic. Moreover, what is not acceptable as a characterization of logic as a whole may nevertheless serve as a heading or slogan for a partisan movement or interesting research program *within* logic. Truth-value semantics and other approaches of ontological minimalism[20] could very well adopt this battle-cry against more ontologically-minded directions in logic: *Logic is the science of the laws of truth*, and of nothing else - or if it is not, let us see to what extent it can be treated in an ontologically neutral manner.

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[20] See Meixner 1995. Concerning truth-value semantics, see Leblanc 1983, pp. 189-274, in particular p. 191.

[21] Translations into English of the cited papers by Frege can be found in: *Collected Papers on Mathematics, Logic, and Philosophy*, ed. B. McGuiness, Oxford: Blackwell 1984; *Posthumous Writings*, eds. H. Hermes et al., Oxford: Blackwell 1979.